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Studies on pressure drop and minimum fluidization velocity of gas-solid fluidization of homogeneous well-mixed ternary mixtures in un-promoted and promoted square bed

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ABSTRACT

This paper reports the experimental finding relating to fluidization characteristics of homogenous wellmixed ternary mixtures of three different particle size at varying compositions. The study has been carried out in an un-promoted as well as a rod-promoted square bed. The bed voidage, fixed bed pressure drop and the minimum fluidization velocity have been obtained for both the above-mentioned beds. The dependence of these quantities on average particle diameter and mass fraction of the fines in the mixture for both types of beds has been discussed. The bed voidage and minimum fluidization velocity have been found to decrease with increase in the mass fraction of fines in the mixture. The experimental values of fixed bed pressure drop have been compared with those predicted from equations available in literature. The Kozney–Carman equation has been found to be significant in the present case. The experimental values of minimum fluidization velocity have been compared with the respective values calculated from the correlations proposed by earlier investigations for mono-size and binary mixture of particles using the Sauter mean diameter for the particle size. The values for minimum fluidization velocity calculated from the equation of Wen and Yu have been found to be close to the experimental values.

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1. Introduction

Fluidization is an established fluid–solid contacting technique, which finds extensive applications in combustion, gasification, carbonization, drying of solids, coating of particles, and many others. The fluidized bed reactor is extremely acclaimed in the process industry for its specific features viz. high rate of heat and mass transfer, continuity in operation and rapid solid mixing leading to nearly isothermal conditions throughout the bed.

In the recent past, fluidized-bed reactors have received much attention in the biotechnology sector, one of the most common applications being in the wastewater treatment. In such bioreactors, the properties (size, density, shape) of the fluidized particles can change during the process because of their colonization by a biofilm. In other industrial applications viz. combustion, gasification, etc., the fluidized particles, though uniform in size at beginning, may change due to attrition, coalescence and chemical reaction thereby affecting the quality of fluidization [1]. Therefore proper characterization of the bed dynamics for the binary and the multi-component mixtures in gas–solid fluidized systems is an important prerequisite for their effective utilization [2]. In gas–solid fluidization, the use of binary-solid fluidized beds for thermochemical processing of biomass is well established. In gas-solid fluidized bed bioreactors, microorganisms are immobilized onto particles which are fluidized by a gas stream [3].

With the development of fluidized bed coal combustion and the recent interest in the use of fluidized beds for waste utilization and for dry solids separation, the potential applications of multi-component fluidized beds are on the rise. The combination of particle size, density and shape that may be found in such fluidized beds is more or less infinite, but great insight into their general behaviour can be found from a study of these mixtures [4]. In many industrial dense gas-fluidized bed processes, e.g., gas phase polymerization and fluidized bed granulation, mixtures of particles with different physical properties are encountered [5]. Fluidized beds are used extensively in the chemical and pharmaceutical industries in processes that include coating of tablets, expanded bed adsorption of biomolecules, fluid catalytic cracking, and incineration of solid waste. In many process industries, fluidization systems use particles (e.g., catalysts or resins) which have a wide distribution of particle sizes [6]. Burning of high sulfur coal in fluidized bed combustor is one of the most promising methods that are environmentally acceptable and economically feasible. In this operation and also in fluidized bed granulation at least a ternary mixture with little or no density differences is experienced [7].

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Nomencla	ture
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d _c	size of coarse particles
$d_{\rm f}$	particle size of fines
$d_{\rm p}$	particle size and average particle size in mixture (m)
$d_{\rm pi}$	particle size of individual components in mixture
	(m)
$d_{\rm p,mm}$	mass mean particle diameter (m)
$d_{\rm p,sm}$	Sauter mean particle diameter (m)
g	acceleration due to gravity (m/s ²)
$G_{\rm f}$	superficial gas mass velocity (kg/h m ²)
G _{mf}	superficial gas mass velocity at minimum fluidiza-
	tion (kg/hm^2)
Н	bed height (m)
Hs	initial static bed height (m)
ΔP	bed pressure drop (Pa)
Ug	superficial linear gas velocity (m/s)
U _{mf}	minimum fluidization velocity (m/s)
U_{mf}^{c}	minimum fluidization velocity of coarse component
1111	(m/s)
U_{mf}^{f}	minimum fluidization velocity of fine component
1111	(m/s)
U^i_{mc}	minimum fluidization velocity of the component
1111	having intermediate size (m/s)
$U_{m_{f}}^{m}$	minimum fluidization velocity of the ternary mix-
mi	ture (m/s)
Xi	mass fraction of individual components, intermedi-
-1	ate size particles in the mixture
Xf	mass fraction of fines in the mixture
. 1	
Greek sy	rmbols
ε	bed voidage
μ_{σ}	viscosity of fluid (kg/m s)
ρ _f	density of fine particles (kg/m^3)
ρσ	density of gas (kg/m^3)
$\rho_{\rm n}$	density of solid (kg/m^3)
$\rho_{\rm sm}$	mean density of solid (kg/m^3)
φ_{s}	average sphericity of solid particles in mixture
φ _{si}	sphericity of particles of each size range in mixture
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The importance of the study of mixture of solids (binary or ternary) of different sizes has been highlighted by many researchers [1–15]. The various aspects of bed dynamics viz. bed pressure drop [1,3,4,6], bed voidage [1], minimum fluidization velocity [3,4], bed expansion [4,6] and bed composition [6] relating to binary mixtures (both homogeneous and heterogeneous) in gas-solid systems have been investigated. Thus the thermo-chemical processing of solids/biomass in gas-solid fluidized bed involving binary mixtures is well established. However, the information available for ternary mixture is meagre [7].

The salient features of a square bed as gas–solid fluidizer with respect to a conventional one have been highlighted by Sahoo and Roy [16] and Singh et al. [17,18]. Further Singh et al. have predicted the minimum fluidization velocity [17] and the bed pressure for such beds with the help of modified Ergun's equation [17,18]. The significant contribution of rod and other type of promoters in improving the fluidization quality for conventional beds has been discussed by several authors [16,19–21]. Sahoo and Roy [16] have studied the effect rod promoter on bed pressure drop in a square gas–solid fluidized bed and have proposed correlation in terms of Euler number for un-promoted and rod–promoted beds. They have developed ANN model and compared the predicted values. In view of the limited information available for ternary mixtures in general and promoted square bed in particular, the present study has been taken up to investigate a few bed parameters viz. fixed bed voidage, pressure drop and minimum fluidization velocity of ternary mixture of particles in un-promoted and rod-promoted square gas–solid fluidized beds.

2. Experimental

A schematic representation of the experimental setup is given as Fig. 1. The experimental setup consists of an air compressor, constant pressure tank, rotameter, silica gel column, $0.08 \text{ m} \times 0.08 \text{ m}$ square cross-section and 0.94 m high Perspex column (fluidizer) with two pressure tapings and a differential U-tube manometer containing carbon tetrachloride as the manometric fluid. Compressed and dried air has been used as the fluidizing medium. The calming section is followed by a GI plate of one mm thickness having 37 nos. of orifices placed in an equilateral triangular pattern at a pitch of 7.5 mm to act as distributor for the uniform entry of air to the fluidizer. A mild steel wire mesh is placed over the distributor to prevent the entry of materials into the calming section. Bed heights have been evaluated by averaging the values read on three graduated scales put at 120° interval around the column wall, and then used for determining the bed void fraction.

The pressure drop through the fixed and fluidized beds have been recorded from the manometer readings with varying operating parameters viz. air flow rate, mixture composition, and initial static bed height. The procedure has been repeated by introducing a rod promoter to the bed. The details of the rod promoter have been presented Fig. 1. The rod promoter has one central rod of 0.006 m Ø and four numbers of radial rods of 0.004 m Ø. The item 10 in Fig. 1 shows the placement of rods and configuration of the promoter. The minimum fluidization velocity has been determined from the plot of bed pressure drop vs. superficial gas mass velocity. The scope of the present investigation has been given in Table 1. Three closely sieved samples of dolomite have been used as the bed material. For ternary mixture fairly good mixing has been achieved by coning and quartering method as done in experimental practice and classification has been avoided since the ratio of the largest to the smallest particle in the mixture was less than 2.3. For all the mixtures constant initial static bed heights have been taken for which the bed mass varies to a negligible extent.

As regards well-mixed mixtures, since it is practically impossible to measure their height right at their minimum fluidization point, where they begin to undergo size segregation accompanied by some bubbling, the experimental dependence of bed porosity on fraction of fines has been assumed to be the same of that of fixed bed porosity. As in the case of mono-disperse beds of particles belonging to Geldart's B group, this approximation introduces a negligible error in calculation, since no significant expansion occurs in the transition from the packed to the incipiently fluidized state.

3. Results and discussion

In all the experiments, the well-mixed arrangement of particles is used as the initial particle bed as was adopted by Huilin et al. [9] and Formisani et al. [10].

3.1. Fixed bed porosity and pressure drop

Fig. 2 presents the experimentally determined fixed bed porosity for both the promoted and un-promoted beds as a function of average particle size calculated for the ternary mixture from

$$d_{\mathrm{p,sm}} = \frac{1}{\sum_{i} (x_i/d_{\mathrm{p}i})} \tag{1}$$

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Fig. 1. Experimental setup.

Table 1Scope of the experiment

(A) Properties of bed material					(B) Ternary mixture properties					
Materials Particle size $(d_p \times 10^3 \text{ m})$		$\rho_{\rm s} ({\rm kg}/{\rm m}^3)$	Particle size ratio	Mixture	Composition, wt% $(d_{p1}: d_{p2}: d_{p3})$	Average particle size $(d_p \times 10^3 \text{ m})$				
Dolomite (d_{p1}) Dolomite (d_{p2}) Dolomite (d_{p3})	0.725 0.55 0.390	2705 2705 2705	$d_{p1}/d_{p2} = 1.318$ $d_{p2}/d_{p3} = 1.41$ $d_{p1}/d_{p3} = 1.859$	Mixture-1 Mixture-2 Mixture-3 Mixture-4	50:50:0 45:45:10 40:40:20 35:35:30	0.625 0.590 0.558 0.530				
(C) Bed parameter	r									
<i>H</i> _s (m)	0.06	0.09	0.12	0.15	$D_{\rm e}$, UP = $D_{\rm c}$ = 0.08 m	<i>D</i> _e , RP = 0.06499 m				

The relation between fixed bed porosity and mass fractions of fine particles in the ternary mixture is given in Fig. 3. It can be seen form this figure that the porosity decreases with the increase of mass fraction of fine particles or decrease of the average particle size of the mixture. The decrease is continuous for the present range of mixture composition studied. With further increase in mass fraction of fines the bed porosity is likely to increase as reported by earlier investigators [9,10].

By using the experimental values of fixed bed porosity, the packed bed pressure drop has been calculated from different available equations and compared with the experimental ones. With increasing gas velocity, the bed pressure drop increases in the fixed bed regime. Once the minimum fluidization condition is reached, with little variation the pressure drop remains constant. The pressure drop in the fixed bed and at minimum fluidization is normally calculated by Ergun's equation or Carman–Kozney equation. Huilin et al. [9] have used the Ergun's equation to predict the pressure drop through the bed with binary mixture of spherical particles. Using the Sauter mean diameter as the average particle size, the Ergun's equation for the ternary mixture of irregular particles can be written as

$$\frac{\Delta P}{H} = \frac{150U_{\rm g}\mu_{\rm g}(1-\varepsilon)^2}{\varphi_{\rm s}^2 d_{\rm p,sm}^2 \varepsilon^3} + \frac{1.75U_{\rm g}^2\rho_{\rm g}(1-\varepsilon)}{\varphi_{\rm s} d_{\rm p,sm} \varepsilon^3}$$
(2)

Formisani et al. [10] using a modified Carman–Kozney equation, have predicted the pressure drop of a well-mixed bed of binary mixture of spherical particles. The prediction was accurate as long as the mixed bed structure was not destroyed, i.e., upto incipient of fluidization. The modified equation for the ternary mixture of irregular particles can be written as

$$\Delta P = 180 \frac{U_g \mu_g (1-\varepsilon)^2}{\varphi_s^2 d_{p,\rm sm}^2 \varepsilon^3} H$$
⁽³⁾

The average sphericity for the ternary mixture has been calculated by two different methods. First by the use of the correlation of Narsimhan [22] for mono-disperse particles. For binary and ternary mixture the equation can be written as

$$\frac{1-\varepsilon}{\varphi_{\rm s}} = 0.231 \log d_{p,\rm sm} + 1.417 \tag{4}$$

where d_p is the average particle diameter in feet. In the second method the average sphericity has been calculated from the sphericity data of irregular particles of dolomite of different sizes as reported by Singh [23]. The average sphericity here has been taken as the mass mean sphericity and has been calculated using the following equation:

$$\varphi_{\rm s} = \sum_{i} x_i \varphi_{\rm si} \tag{5}$$



Fig. 2. Variation of fixed bed porosity with average particle size.



Fig. 3. Variation of fixed bed porosity with fraction of fines in the mixture.

The average sphericity calculated by Eq. (5) from sphericity data for irregular particles as reported by Singh [23] has been correlated which is given by

$$\frac{1-\varepsilon}{\varphi_{\rm s}} = -0.212 \ln d_{p,\rm sm} - 0.822 \tag{6}$$

This equation can be used to predict the average sphericity of such ternary mixtures.

Figs. 4 and 5 show the pressure drop in fixed bed for the binary and a ternary mixture, respectively. It is clear from Fig. 4 that the values of fixed bed pressure drop calculated from Eqs. (2) and (3)are close to the experimental values when sphericity is calculated from Eq. (6). The fixed bed pressure drop values calculated by Eqs. (2) and (3) with sphericity obtained from Eq. (4) show higher deviation from the corresponding experimental ones. Nevertheless for all the cases the calculated fixed bed pressure drop overpredicts the values than the experimental ones. Hence for all the rest of the pressure drop calculations, the sphericity has been obtained from Eq. (6) and used. Fig. 5 presents the variation of fixed bed pressure drop with superficial gas mass velocity upto the condition of incipient fluidization for the ternary mixture of composition 40:40:20. The deviation of the calculated bed pressure drop values from Eqs. (2), (3) and (6) is more, around the point of minimum fluidization. At low gas mass velocity the values are close but as the gas mass velocity increases the deviation increases. Both the equations predict higher values of fixed bed pressure drop than the experimental ones. However, Eq. (2) predicts lower values of bed pressure drop than the values calculated by Eq. (3) and close to the experimental ones. In Eq. (3) the coefficient is 180 where as in original Kozney-Carman equation this value is 150. Thus the original Kozney-Carman equation as represented by Eq. (7) has also been



Fig. 4. Variation of fixed bed pressure drop with superficial gas mass velocity for binary mixture of composition 50:50:00 in un-promoted bed.



Superficial gas mass velocity for ternary mixture, kg/hr.m²

Fig. 5. Variation of fixed bed pressure drop with superficial gas mass velocity for ternary mixture of composition 40:40:20 in un-promoted bed.

Comparison	of fixed b	ed pressure	drop

Table 2

considered for bed pressure drop measurement:

$$\Delta P = 150 \frac{U_g \mu_g (1-\varepsilon)^2}{\varphi_s^2 d_{p,\text{sm}}^2 \varepsilon^3} H$$
⁽⁷⁾

The fixed bed pressure drop predicted from Eqs. (2), (3), (6) and (7) have been compared in Table 2. For all cases Eqs. (2) and (3) predict higher values of pressure drop than the experimentally measured ones. It is also evident from Table 2 that the original Kozney–Carman equation, i.e., Eq. (7) is the best fit to the experimental values.

3.2. Minimum fluidization velocity

The analysis of the fluidization characteristics of a binary or a ternary mixture undergoing segregation by size presents, as a first difficulty, the problem of adopting a proper definition of its minimum fluidization velocity. As in the case of mono-disperse beds, the parameter U_{mf} not only quantitatively indicates the amount of drag force needed to attain solid suspension in the gas phase, but also constitutes a reference for the evaluation of the intensity of the fluidization regime at higher velocity levels. Owing to the fact that the onset of fluidization is always accompanied by that of a segregation phenomenon, the definition of the minimum fluidization velocity of a binary or a ternary mixture is not obvious, and has to be discussed in the light of the specific characteristics of the system. In particulate systems formed by two narrow cuts of spheres of the same material, percolation of the fine component through the packed bed of the coarse one cannot take place if the diameter ratio d_f/d_c is higher than a limiting value that depends on the degree of looseness of the coarse packing. Simple geometric considerations indicate that the theoretical limit for interstitial penetration of fines is given by $d_f/d_c = 0.41$, that corresponds to the most open, cubic coarse structure [10]. However, in all practical cases the coarse sphere packing is random and consequently the

Type of mixture	$H_{\rm s}\left({\rm m} ight)$	$d_{\rm p}\times 10^3~(\rm m)$	$U_{\rm g}({\rm m/s})$	$\Delta P(\text{Exp})$	ΔP from Eq. (2)	Deviation in (%)	ΔP from Eq. (3)	Deviation in (%)	ΔP from Eq. (7)	Deviation in (%)
50:50:00	0.12	0.625	0.061	239.5	309.0	29.0	354.7	48.1	295.6	23.4
			0.114	581.6	595.2	2.33	658.7	13.2	548.9	-5.63
			0.175	872.5	954.1	9.35	1013.4	16.1	844.5	-3.21
			0.200	1060.7	1110.8	4.72	1160.5	9.41	967.1	-8.82
			0.226	1231.8	1271.8	3.24	1307.3	6.13	1089.4	-11.5
			0.256	1420.0	1472.4	3.69	1484.6	4.55	1237.2	-12.9
			0.287	1642.4	1679.8	2.27	1662.0	1.19	1385.0	-15.7
			0.310	1762.1	1838.1	4.31	1793.8	1.79	1494.8	-15.2
45:45:10	0.12	0.590	0.061	431.6	405.1	6.14	466.6	8.10	388.8	-9.91
			0.114	828.7	778.3	6.09	866.5	4.56	722.1	-12.8
			0.175	1243.1	1243.9	0.06	1333.1	7.24	1110.9	-10.6
			0.200	1467.5	1446.6	1.43	1526.6	4.02	1272.2	-13.3
			0.226	1622.9	1654.4	1.93	1719.7	5.96	1433.1	-11.7
			0.256	1726.5	1912.9	10.8	1953.0	13.1	1627.5	-5.74
			0.287	1830.1	2179.5	19.1	2186.3	19.5	1821.9	-0.45
40:40:20	0.12	0.558	0.061	483.4	501.2	3.68	578.8	19.7	482.4	-0.22
			0.114	915.0	960.9	5.00	1074.9	17.5	895.8	-2.10
			0.175	1381.2	1532.1	10.9	1653.8	19.7	1378.2	-0.22
			0.200	1553.9	1780.2	14.6	1893.9	21.9	1578.2	1.56
			0.226	1726.5	2034.1	17.8	2133.4	23.6	1777.8	2.97
			0.256	1881.9	2349.5	24.8	2422.8	28.7	2019.0	7.28
35:35:30	0.12	0.530	0.061	508.5	613.3	20.6	709.95	39.7	591.6	16.3
			0.114	1017.1	1173.5	15.4	1318.5	29.6	1098.7	8.02
			0.175	1474.7	1867.4	26.6	2028.4	37.5	1690.3	14.6
			0.200	1695.1	2167.9	27.9	2322.8	37.0	1935.7	14.2
			0.226	1898.5	2475.2	30.4	2616.6	37.9	2180.5	14.8
Standard deviation						11.02		13.47		11 22
Mean deviation						11.02		18 20		0.36
ivicali deviation						11.02		10.29		9.50



Fig. 6. Variation of bed pressure drop with superficial gas mass velocity for ternary mixture of composition 40:40:20 at different *H*_s in un-promoted bed.

 d_f/d_c value that fixes the percolation limit is somewhat lower and has to be determined experimentally. Whenever the fine-to-coarse size ratio is sufficiently high to exclude the possibility of fine percolation, the fluidization pattern of the mixture is found to be strongly affected by the axial distribution of either solid, so that its minimum fluidization velocity cannot be defined on an absolute basis, but only with reference to a particular state of mixing of the two/three component system.

In Figs. 6–9, the nature of the pressure drop curve with gradual increase in the superficial gas velocity indicates that the mixture is more or less well mixed as reported by Huilin et al. [9] and Formisani et al. [10]. For a segregated bed different growth pattern of pressure drop is seen. The experimental minimum fluidization velocity is normally assumed as the superficial gas velocity corresponding to the intersection of the fixed bed pressure drop line with the horizontal line representing the fluidized state. Whatever be its average composition, when the mixture is charged as a homogeneous assembly of particles, its fixed bed voidage as well as its minimum fluidization voidage are lower than that of each of its single components [10]. For this kind of system, the onset of the fluidized state is gradual: a front of fluidization is observed which travels from the top to the bottom of the particulate bed, starting at a velocity always intermediate to $U_{\rm mf}$ values of its single components.

Fig. 6 shows the variations of bed pressure drop in both fixed and fluidized bed regimes with the superficial gas velocity for different initial static bed heights. From this it is clear that the measured fluidized bed pressure drop agrees well with the bed weight per unit column cross-section. This proves the correctness of the measurement of pressure drop and also it indicates that there is no wall effect. The intersection of the experimental fixed and fluidized bed pressure drop curves has been considered as the point of minimum



Fig. 7. Variation of bed pressure drop with superficial gas mass velocity for ternary mixture at $H_s = 0.12$ m for different mixture compositions in un-promoted bed.



Fig. 8. Variation of bed pressure drop with superficial gas mass velocity for ternary mixture at $H_s = 0.12$ m for different mixture compositions in rod-promoted bed.



Fig. 9. Variation of bed pressure drop with superficial gas mass velocity for ternary mixture of different compositions at $H_s = 0.12$ m in un-promoted bed.

fluidization. For all the initial static bed heights, the bed has been found to attain the minimum fluidization condition at the same superficial gas mass velocity. Thus it can be concluded that there is no effect of bed mass (i.e., initial static bed height) on the minimum fluidization velocity.

The minimum fluidization condition has been taken to be the intersection of experimental fixed and fluidized bed pressure drop curves for the ternary mixtures of different composition in both the un-promoted and the rod-promoted beds. Figs. 7 and 8 show the variation of bed pressure drop with superficial gas mass velocity for ternary mixtures in both the un-promoted and the rod-promoted bed, respectively. For both the cases the fluidized bed pressure drop is found to increase with increase in fines in the ternary mixtures. This is due to the decrease in bed voidage and increased bed mass for the same initial static bed height. A gradual decrease in minimum fluidization gas mass velocity with increase in fines for the ternary mixtures has been observed in both the un-promoted and the rod-promoted beds. For all mixtures, the experimental minimum fluidization velocity is found to be more in the rod-promoted bed than in an un-promoted one. The possible reason may be due to a fraction of the fluid flowing upward through the channel along the rods of the promoter rather than in the actual bed of particles. In true application promoters are used to decrease the bed fluctuation by rupturing large gas bubbles in the fluidization regime. Fig. 9 represents the variation of fixed bed pressure drop (experimental and those calculated from Eqs. (2) and (7)) with the superficial gas mass velocity. For mixture composition 35:35:30, the Kozney-Carman equation under predicts the minimum fluidization gas mass velocity than that obtained experimentally. Whereas the minimum fluidization gas mass velocity is overpredicted for the binary mixture of composition 50:50:00, for all the ternary mixtures the predicted values of the minimum fluidization



Fig. 10. Comparison of minimum fluidization velocities for ternary mixture in unpromoted bed.

gas mass velocity are close to the experimental ones. Ergun's equation accurately predicts the minimum fluidization gas mass velocity of binary mixture. But the minimum fluidization gas mass velocities of ternary mixtures are underpredicted by Ergun's equation.

The minimum fluidization gas velocity from experiment and those predicted by some existing correlations (for binary mixtures and mono-size particles) have been compared in Figs. 10 and 11 for the un-promoted and the rod-promoted beds, respectively. The original correlations of Bilbao et al. and Chiba et al. as reported by Clarke et al. [3] for completely mixed bed of homogeneous binary mixture of particles are given Eqs. (8) and (9), respectively:

$$U_{\rm mf}^{\rm m} = U_{\rm mf}^{\rm c} - (U_{\rm mf}^{\rm c} - U_{\rm mf}^{\rm f})X_{\rm f} \tag{8}$$

where $X_{\rm f}$, the real volume fraction of fines in the binary mixture is same as the mass fraction of fines in the mixture for different particles of same density:

$$U_{\rm mf}^{\rm m} = U_{\rm mf}^{\rm f} \frac{\bar{\rho}}{\rho_{\rm f}} \left(\frac{\bar{d}}{d_{\rm f}}\right)^2 \tag{9}$$

where $\bar{\rho}$ and \bar{d} are the mean values of particle density and the particle size, respectively, based on volume fractions. For mixture of particles of same density \bar{d} is simply the mass mean particle size $d_{p,mm}$. Eq. (8) for ternary mixture of particles can be written as

$$U_{\rm mf}^{\rm m} = U_{\rm mf}^{\rm c} - (U_{\rm mf}^{\rm c} - U_{\rm mf}^{\rm i})x_i - (U_{\rm mf}^{\rm c} - U_{\rm mf}^{\rm f})x_f$$
(10)

Eq. (10) has been used to calculate the minimum fluidization velocity for the homogeneous ternary mixture. The minimum fluidization velocity of individual particles has been calculated from equation of Wen and Yu [24] given by

$$U_{\rm mf} = \frac{\mu_{\rm g}}{d_{\rm p}\rho_{\rm g}} \left\{ \left[(33.7)^2 + 0.0408 \frac{d_{\rm p}^3 \rho_{\rm g} (\rho_{\rm p} - \rho_{\rm g}) g}{\mu_{\rm g}^2} \right]^{0.5} - 33.7 \right\} (11)$$

Type of mixture	$d_{\rm p} \times 10^3 \ ({\rm m})$	Minimum fluidization velocity of mixture, m/s and percentage deviation from experimental								
		Exp	Eq. (7)		Eq. (9)		Eq. (10)		Eq. (11)	
Un-promoted bed										
50:50:00	0.625	0.30977	0.38177	23.2	0.35893	15.8	0.32185	3.90	0.31257	0.90
45:45:10	0.590	0.28702	0.31686	10.4	0.31922	11.2	0.30362	5.78	0.28512	-0.66
40:40:20	0.558	0.25639	0.27118	5.77	0.28575	11.4	0.28539	11.3	0.26077	1.70
35:35:30	0.530	0.22576	0.23344	3.40	0.25728	13.9	0.26716	18.3	0.23913	5.92
Rod-promoted bed										
50:50:00	0.625	0.32115	0.40726	26.8	0.35893	11.7	0.32185	0.22	0.31257	-2.67
45:45:10	0.590	0.29839	0.33803	13.3	0.31922	6.98	0.30362	1.75	0.28512	-4.44
40:40:20	0.558	0.27171	0.28889	6.32	0.28575	5.17	0.28539	5.17	0.26077	-4.02
35:35:30	0.530	0.24108	0.26020	7.93	0.25728	6.72	0.26716	6.72	0.23913	-0.81

Table 3	
Comparison of minimum fluidization velocit	y

In using Eq. (9) to calculate the minimum fluidization velocity for the ternary mixture in an un-promoted bed, both mass mean particle diameter, $d_{p,mm}$ and Sauter mean particle diameter, $d_{p,sm}$ have been used. The fine component minimum fluidization velocity, U_{mf}^{f} has been calculated from Eq. (11). From Fig. 10 it is seen that using $d_{p,mm}$, Eq. (9) predicts the values of the minimum fluidization velocity for ternary mixtures much higher than the experimental ones, while the use of $d_{p,sm}$ in the same equation predicts the values closer to the experimental. However, by use of the mass mean particle size or the Sauter mean particle size, Eq. (9) predicts the values of the minimum fluidization velocity of ternary mixtures higher than the experimental ones. In subsequent calculation of the values of the minimum fluidization velocity for the ternary mixtures Sauter mean diameter has been used as this gives a better fit.

Eq. (10) predicts the minimum fluidization velocity of the binary mixture of composition 50:50:00 (ternary mixture with zero fines) nearly the same as that obtained form the experiment in case of both the un-promoted and the rod-promoted beds as shown in



Fig. 11. Comparison of minimum fluidization velocities for ternary mixture in rodpromoted bed.

Figs. 10 and 11, respectively. With increase in fines in the ternary mixtures the predicted values of the minimum fluidization velocity for the ternary mixtures have been found to deviate more and more from the experimental ones. With increase in fines the predicted values are higher than the experimental ones. The original equation of Kozney-Carman (Eq. (7)) shows a better prediction of fixed bed pressure drop over the others. Thus the values of the minimum fluidization velocity for ternary mixtures in both the un-promoted and the rod-promoted beds have been predicted from Eq. (7) and compared in Figs. 10 and 11, respectively. In calculating the values of the minimum fluidization velocity from Eq. (7), the experimental values of the bed pressure drop and the bed voidage at the minimum fluidization have been used. This predicts higher values of the minimum fluidization velocity for the ternary mixtures with fewer fines. As the fraction of the fines increases in the mixture, the predicted values of the minimum fluidization velocity drastically reduce. For ternary mixture of composition 35:35:30, the predicted minimum fluidization velocity is very close to the experimental one

The minimum fluidization velocity values have also been predicted form Eq. (11) using the Sauter mean particle size and the values obtained for ternary mixtures of varying compositions in both the un-promoted bed and the rod-promoted beds have been compared in Figs. 10 and 11, respectively. For both the cases very close agreement has been found between the predicted and the experimental values of the minimum fluidization velocity. Thus the equation of Wen and Yu [24] can be used for predicting the minimum fluidization velocity for such ternary mixtures in case of both the un-promoted bed and the rod-promoted square bed. Table 3 shows the values of minimum fluidization velocity (both experimental and those predicted from various correlations) and the percentage deviations (mean and standard) of the predicted values from the experimental. It is seen that nearly most of the values are within 15% deviation. This result is quite significant and indicates that the predictions are not very far from the experimental values.

4. Conclusions

The fluidization behaviour of a ternary mixture differing in particle sizes with the same density is strongly influenced by the variations of average particle diameter and mass fraction in the bed. The average particle diameter, the mass fraction of particles of different sizes and the bed voidage considerably affect the fixed bed pressure drop and the onset of fluidization.

In view of the closeness of the values to the experimental ones, the fixed bed pressure drop values for ternary mixtures can be calculated by the Kozney–Carman equation (Eq. (7)) with fairly good accuracy over the range of the present investigation. For the calculation of the values of minimum fluidization velocity for ternary mixtures, the equation of Wen and Yu [24] (Eq. (11)) can be used with fairly good accuracy as compared to the equations of Chiba et al. (Eq. (9)), Bilbao et al. (Eq. (10)) and Kozney–Carman (Eq. (7)). Both Eqs. (7) and (9) can be used to calculate the values of the minimum fluidization velocity of the ternary mixtures with higher percentage of fines. Eq. (10) is quite accurate in predicting the minimum fluidization velocity of ternary mixture in a rod-promoted bed. These predictions will provide information useful for the design of gas–solid fixed and fluidized bed ternary systems with potential application for combustion, gasification and solid catalyzed chemical and bio-chemical reactions.

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